Math 115

Winter 2017

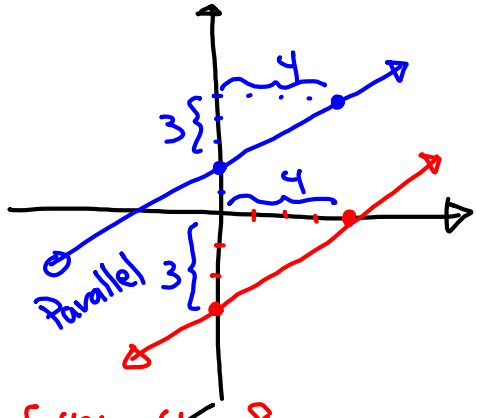
Lecture 10

Class Quiz

① Solve by graphing:

$$\begin{cases} 3x - 4y = 12 \\ y = \frac{3}{4}x + 2 \end{cases}$$

$\boxed{\emptyset}$



② Solve: $\begin{cases} 2x + 3y = -4 \\ 3x + 2y = -1 \end{cases}$

$\times 2$, $\times 3$

$$\Rightarrow \begin{cases} -4x - 6y = 8 \\ 9x + 6y = -3 \end{cases}$$

$5x = 5 \Rightarrow \boxed{x = 1}$

$3(1) + 2y = -1$

$3 + 2y = -1$

$2y = -4$

$\boxed{y = -2}$

$\boxed{(1, -2)}$

① Find eqn of a line that contains (2,0) & (0,-3).

$m = \frac{0-(-3)}{2-0} = \frac{3}{2}$

$y - y_1 = m(x - x_1)$

$y - -3 = \frac{3}{2}(x-0)$

$y+3 = \frac{3}{2}x$

$\boxed{y = \frac{3}{2}x - 3}$

② Find eqn of a line that contains (3,-2) and Parallel to $2x-3y=5$.

$-3y = -2x+5$

$y = \frac{-2}{-3}x + \frac{5}{-3}$

$m = \frac{2}{3}$ → Same slope

$y-(-2) = \frac{2}{3}(x-3)$

$y+2 = \frac{2}{3}x - 2$

$\boxed{y = \frac{2}{3}x - 4}$

③ Find eqn of a line that contains (-4, 1) with

a) Zero slope

Horizontal line

$y = b$

$\boxed{y=1}$

b) undefined slope

Vertical → x-only → $x = a$

$\boxed{x=-4}$

c) slope 2

$y - y_1 = m(x-x_1)$

$y-1 = 2(x--4)$

$y-1 = 2(x+4)$

$y-1 = 2x+8$

$\boxed{y = 2x+9}$

d) perpendicular to $y = \boxed{4}x - 5$.

$m = \frac{-1}{4}$

$y - y_1 = m(x-x_1)$

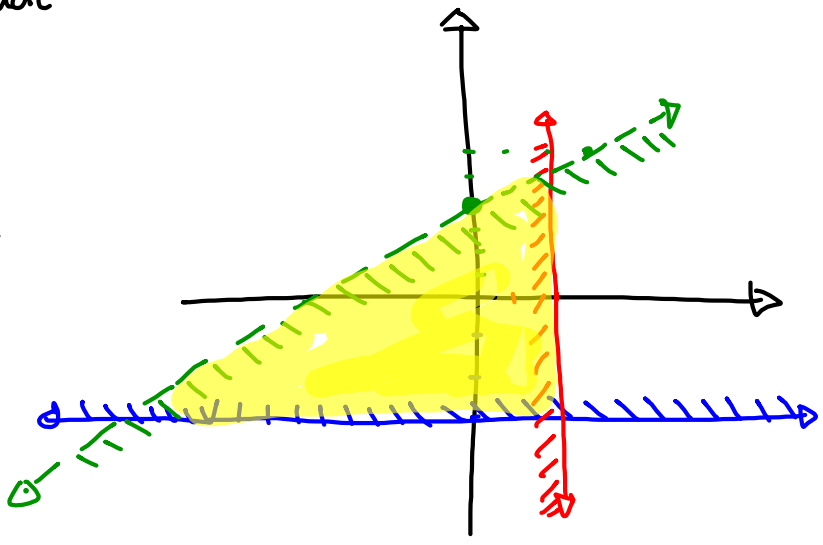
$y-1 = \frac{-1}{4}(x--4)$

$y-1 = \frac{-1}{4}x - 1$

$\boxed{y = \frac{-1}{4}x}$

Graph & shade

$$\begin{cases} y \geq -3 \\ x \leq 2 \\ y < \frac{2}{3}x + 4 \end{cases}$$



John has \$2.05 in Quarters & Dimes only.

# of quarters is 1 more than twice # Dimes

How many of each?

D → Dimes

R → Quarters

$$\begin{cases} 10D + 25R = 205 \\ R = 2D + 1 \end{cases}$$

$10D + 25(2D+1) = 205$

$10D + 50D + 25 = 205$

$60D = 180$

$D = 3$

$R = 7$

3 Dimes

7 Quarters

Two angles are complementary.
The sum of 3 times one of them and twice the other one is 200°.
Find the larger angle.

$$\begin{cases} x + y = 90 \\ 3x + 2y = 200 \end{cases} \quad -2$$

$$x = 20$$

$20 + y = 90$

$y = 70$

Larger angle is 70°

Two angles are supplementary.
5 times one of them is equal to 4 times the other one.
Find both angles.

$$\begin{cases} x + y = 180 \\ 5x = 4y \end{cases}$$

$$4 \begin{cases} x + y = 180 \\ 5x - 4y = 0 \end{cases} \Rightarrow \begin{cases} 4x + 4y = 720 \\ 5x - 4y = 0 \end{cases}$$

$9x = 720 \Rightarrow x = 80$

$80 + y = 180 \rightarrow y = 100$

80° & 100°

We need 100 liters of 36% alcohol Soln.

We have unlimited supply of 30% & 40% alcohol Soln. How many liters of each do we need?

30%	+	40%	=	36%
x		y		100

$$\begin{cases} x + y = 100 \\ \frac{30}{100}x + \frac{40}{100}y = \frac{36}{100}(100) \end{cases}$$

$$\begin{cases} x + y = 100 \\ 30x + 40y = 3600 \end{cases} \quad \div 10$$

$$\begin{cases} x + y = 100 \\ 3x + 4y = 360 \end{cases} \quad -3$$

$$y = 60$$

60 L of 40%
&
40 L of 30%

We have unlimited supply of 30% & 60% acid Soln.

We need 30 L of 50% acid Soln.

How many liters of each?

30%	+	60%	=	50%
x		y		30

$$\begin{cases} x + y = 30 \\ \frac{30}{100}x + \frac{60}{100}y = \frac{50}{100} \cdot 30 \end{cases} \quad 100$$

$$\begin{cases} x + y = 30 \\ 30x + 60y = 1500 \end{cases} \quad \div 10$$

$$\begin{cases} x + y = 30 \\ x + 2y = 50 \end{cases} \quad -1$$

$$y = 20$$

$$\begin{cases} x + y = 30 \\ 3x + 6y = 150 \end{cases} \quad \div 3$$

So
20 L of 60%
&
10 L of 30%

Class Quiz

① Find eqn of a line that contains $(-4,0)$ & $(0,8)$.

$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{8-0}{0-(-4)} = \frac{8}{4} = \boxed{2}$

$y = mx + b$

$\boxed{y = 2x + 8}$

$y - y_1 = m(x - x_1)$

$y - 8 = 2(x - 0)$ → $\boxed{y = 2x + 8}$

② Find eqn of a line that contains the origin and perpendicular to $2x - 3y = 6$.

$-3y = -2x + 6$

$y = \frac{2}{3}x - 2$

$m = \frac{-3}{2}$

$y - 0 = \frac{-3}{2}(x - 0)$

$\boxed{y = \frac{-3}{2}x}$

Use exponential Rules to Simplify

① $\left(\frac{-2}{3}\right)^0 = \boxed{1}$

$x^0 = 1, \; x \neq 0$

② $\left(\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \boxed{\frac{8}{27}}$

$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$

③ $x^{12} \cdot (x^{4})^{-3}$

$= x^{12} \cdot x^{-12}$

$= x^0 = \boxed{1}$

④ $\frac{-15x^8y^{-2}}{3x^{-4}y^3}$

$= \frac{-15 x^8 x^4}{3 y^3 y^2}$

$= \boxed{\frac{-5x^{12}}{y^5}}$

⑤ $(-5x^4y^2)^3$

$= (-5)^3(x^4)^3(y^2)^3$

$= \boxed{-125x^{12}y^6}$

⑥ $\left(\frac{2x^6}{y^5}\right)^4$

$= \frac{2^4(x^6)^4}{(y^5)^4} = \boxed{\frac{16x^{24}}{y^{20}}}$

⑦ $\left(\frac{1}{2}\right)^{-1}$

$= \left(\frac{2}{1}\right)^1 = \frac{2}{1} = \boxed{2}$

⑧ $\frac{(x^7)^3 \cdot (x^{-4})^5}{(x^{-6})^5 \cdot (x^{-4})^{-1}}$

$= \frac{x^{21} \cdot x^{-20}}{x^{-30} \cdot x^{28}} = \frac{x^{21+(-20)}}{x^{-30+28}}$

$= \frac{x^1}{x^{-2}} = x^{1-(-2)} = \boxed{x^3}$

Distribute & Simplify

① $-5(2x^2+3x-4)+15x-21$

$= -10x^2 - 15x + 20 + 15x - 21 = \boxed{-10x^2 - 1}$

② $2x^2(3x-4) - 6x^3 - 8x^2$

$= 6x^3 - 8x^2 - 6x^3 - 8x^2 = \boxed{-16x^2}$

Find the area

① $A = LW$

$4x^3y$

$10x^4y^2$

$A = (10x^4y^2)(4x^3y)$

$A = 40x^7y^3$

② $A = S^2$

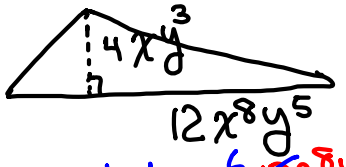
$4x^6y^5$

$A = (4x^6y^5)^2$

$= (4^2)(x^6)^2(y^5)^2$

$A = 16x^{12}y^{10}$

③



$A = \frac{bh}{2} = \frac{6 \cdot 12x^8y^5 \cdot 4xy^3}{2}$

$= 24x^9y^8$

Use FOIL to multiply

① $(3x - 2)(2x - 3)$

$= 6x^2 - 9x - 4x + 6$

$= 6x^2 - 13x + 6$

② $(3x^2 + 4)(3x^2 - 4)$

$= 9x^4 - 12x^2 + 12x^2 - 16$

$= 9x^4 - 16$

③ $(5x + 3)(25x^2 - 15x + 9)$

$= 125x^3 - 75x^2 + 45x + 75x^2 - 45x + 27$

$= 125x^3 + 27$

Due Monday:

SG12, WP 8, 9, and 10

Agenda for Monday:

Exam 2: 6:00 – 7:35

Come early (5:50) if you want extra time

Cumulative exam

Formulas will be provided.

Finish ch.4

Monomial: number times variables that are raised to whole number exponent.

$5x^2$, $-3x^3y^2$, $\frac{2}{3}x^6y^4z^2$, 2017

Coef.

the exponent or sum of the exponent is called degree

when there is no variable → Constant

$-8x^3$ → Monomial
→ Coef. $= -8$
→ Degree $= 3$

$\frac{3}{4}x^7y^5$ → Monomial
→ Coef. $= \frac{3}{4}$, Deg. $= 7+5 = \boxed{12}$

Constant term (Monomial) has degree zero.

ex:

① $-7x$ → Coef. $=-7$, Deg. $=1$

② 1337 → Constant, Deg. $=0$

③ $\frac{4}{7}x^6y^3z$ → Monomial

Coef. $=\frac{4}{7}$, Deg. $=6+3+1=\boxed{10}$

Binomial → Two monomials are separated by + or −.

$4x+7$, $6x^2-12x$, $-3x^2y^3+11xy$

$7x^4-4x^7$

Trinomial → Three monomials are separated by + or −.

x^2-2x+1, $\frac{1}{2}x^6+\frac{2}{3}x^4-\frac{3}{4}x^2$

$12x^2y^3z^4-8x^3y^2z^4+2017$

When many monomial are separated from each other by + or − , we have a Polynomial.

$4x^5 - 2x^3 + 17x^2 - 8$

$-2x^6 - 8x^5 + 12x^4 - 3x^3 + 9x^2 - 100x + 1$

It is recommended to write polynomials in descending order. exponents decrease as you move to the right.

Consider $5x^4 - 27x^3 + 12$

Monomials	Coef.	Degree
$5x^4$	5	4
$-27x^3$	−27	3
12	Constant	0

For the entire trinomial

Deg. 4,

Lead. Coef 5

$32x^8y^3 - 45x^6y^2 + 100xy - 2017$

Monomial	Coef.	Degree
$32x^8y^3$	32	$8+3=11$
$-45x^6y^2$	-45	$6+2=8$
$100xy$	100	$1+1=2$
-2017	Constant	0

Deg. = 11
L.C. = 32
Const. 0

Polynomial

Simplify: $(3x^7)^3 = (-3)^3(x^7)^3$

$= -27x^{21}$

Monomial

Coef. $= -27$, Deg. $= 21$

Simplify: $(2x^2y^6)^4 \cdot 5x^3y$

$= 2^4(x^2)^4(y^6)^4 \cdot 5x^3y$

$= 16x^8y^{24} \cdot 5x^3y$

$= 80x^{11}y^{25}$

Monomial

Coef. 80

Deg 36

Simplify: $\dfrac{-48x^{15}y^{3}}{4x^{3}y^{-3}}$

$= -12x^{15-3}y^{3-(-3)}$

$= -12x^{12}y^{6}$

Monomial

Coef. -12

Deg. $= 18$

Simplify: $-2x^{3}(3x^{2}+5x+1)+2x^{3}$

$= -6x^{5}-10x^{4}-2x^{3}+2x^{3}$

$= -6x^{5}-10x^{4}$ Binomial, Deg. $= 5$, L.C. $= -6$

Simplify

$4x^{2}y^{3}(-2xy^{2}+x^{3}y^{4}-3x^{3}y)$

$= -8x^{3}y^{5} + 4x^{5}y^{7} - 12x^{5}y^{4}$

D=8 D=12 D=9

Trinomial, Deg. $= 12$, L.C. $= 4$

Simplify

$(3x^2+7)(2x^2-3)$

$= 6x^4 - 9x^2 + 14x^2 - 21$

$= 6x^4 + 5x^2 - 21$

Trinomial
Deg. 4
L.C. 6
Const: -21

Simplify

$(7x^5-3x^2)(7x^5+3x^2)$

$= 49x^{10} + 21x^7 - 21x^7 - 9x^4$

$= 49x^{10} - 9x^4$

Binomial
Deg. = 10
L.C. = 49

Scientific Notation

$N \times 10^{n}$ (n an integer)

$1 \le N < 10$

2.5×10^{12}, 1.75×10^{-18}, 6.25×10^{-30}

8.2×10^{25}

we use S.N. for Large or Small numbers.

75000000000

$= 7.5 \times 10^{11}$

0.00000000425

$= 4.25 \times 10^{-9}$

185000000

$= 1.85 \times 10^{8}$

$.000000000005$

$= 5 \times 10^{-12}$

Simplify:

$(2.5 \times 10^{18}) \cdot (3.2 \times 10^{11})$

$= 8 \times 10^{29}$

Simplify: $(8.6 \times 10^{-13}) \cdot (9.5 \times 10^{-17})$

$= 81.7 \times 10^{-30}$

$= 8.17 \times 10^{1} \times 10^{-30} = \boxed{8.17 \times 10^{-29}}$

Simplify $\dfrac{9 \times 10^{-5}}{4.5 \times 10^{15}}$

$= 2 \times 10^{-5-15} = \boxed{2 \times 10^{-20}}$

Simplify: $\dfrac{2.8 \times 10^{18}}{8 \times 10^{-12}}$

$= .35 \times 10^{18-(-12)} = .35 \times 10^{30}$

$= 3.5 \times 10^{-1} \times 10^{30}$

$= \boxed{3.5 \times 10^{29}}$

Simplify:

$$\frac{(7.2 \times 10^{14})(1.5 \times 10^{10})}{3.6 \times 10^{-24}}$$

$$= 3 \times 10^{14+10-(-24)} = \boxed{3 \times 10^{48}}$$

Simplify $\dfrac{2.1 \times 10^{-14}}{8.4 \times 10^{20}}$ $= .25 \times 10^{-34}$

$= 2.5 \times 10^{-1} \times 10^{-34}$

$= \boxed{2.5 \times 10^{-35}}$

① $2.1 \div 8.4 = .25$

② $\dfrac{10^{-14}}{10^{20}} = 10^{-14-20} = 10^{-34}$